

Operations and Algebraic Thinking

Separating Problems

When we begin with a group and separate, or take away, a portion of that group and count up what is left, we are subtracting. The separating method of subtraction is an important beginning strategy that will help children as they develop fluency with single-digit number combinations.

Objective

Solve separating problems by taking away one group from a larger group and counting what is left.

Common Core State Standards

- **K.OA.1** Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
- **K.OA.2** Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

Try It! Perform the Try It! activity on the next page.

Talk About It

Discuss the Try It! activity.

- **Ask:** After you take away from a number, is the number you are left with more or fewer? How do you know?
- **Say:** Look at your Take-Away Workmat (BLM 6). Think about how you use your workmat to solve a take-away problem. **Ask:** What do you put in the big circle? What do you put in the small circle? How do you find the number that is left?

Solve It

With children, reread the problem. Have children reuse the Take-Away Workmat from the Try It! activity. Have children make a drawing to show the number of Three Bear Family® Counters the teacher took away (2 bears) from the 5 bears on the shelf in the small (take-away) circle. Then have children draw the bears that were left on the shelf in the big circle. Encourage children to use their drawings to explain how they solved the problem.

More Ideas

For other ways to teach about subtraction—

- Have the class work together to use Frog Counters or other counters to compose and then solve a separating story problem.
- Have pairs of children place 5 Frog Counters in a paper bag. Instruct one child to take away some of the frogs from the bag. Have children count to find the number of frogs that were taken away. Then encourage children to work together to figure out how many frogs are left in the bag. Finally, have children check to see if they were correct by counting the frogs left inside the bag. Encourage children to complete the following sentence: [number] frogs take away [number] frogs is [number] frogs. For example, 5 frogs take away 3 frogs is 2 frogs. Repeat the activity with different numbers of frogs.

Formative Assessment

Have children try the following problem.

Jamal brought 3 stickers to school to share with his friend Mark. Jamal gave 1 sticker to Mark. Draw a picture to show how many stickers Jamal had left.

Try It! 20 minutes | Independent

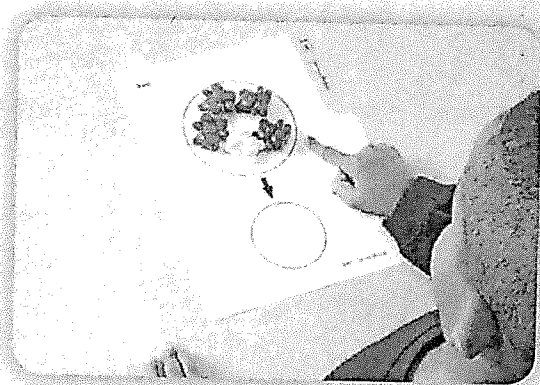
Here is a separating problem.

Rico's teacher put 5 teddy bears on the shelf. Then she took 2 teddy bears away and put them in a basket. How many teddy bears were left on the shelf?

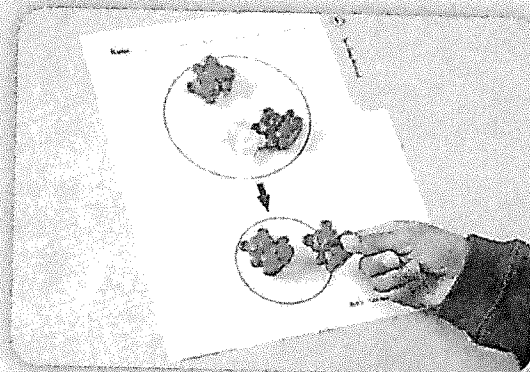
Introduce the problem. Then have children do the activity to solve the problem. Distribute 1 copy of the Take-Away Workmat (BLM 6) and 5 Three Bear Family Counters to each child.

Materials

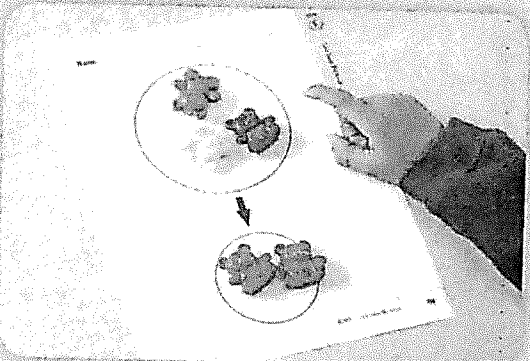
- Three Bear Family® Counters (5 per child)
- Take-Away Workmat (BLM 6; 1 per child)



1. Instruct children to place all of their bears in the big circle on the Take-Away Workmat. Have children count the bears aloud with you one at a time and identify that there are 5 bears in all in the big circle.



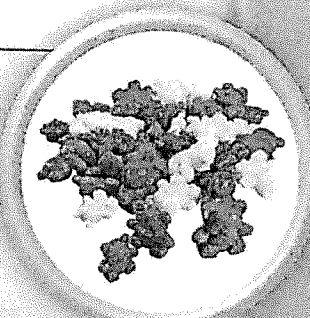
2. Have children take 2 bears away from the big circle and follow the arrow to move them into the small circle.



3. Instruct children to count the bears that are left in the big circle aloud with you. Then help children to understand that 5 take away 2 is 3.

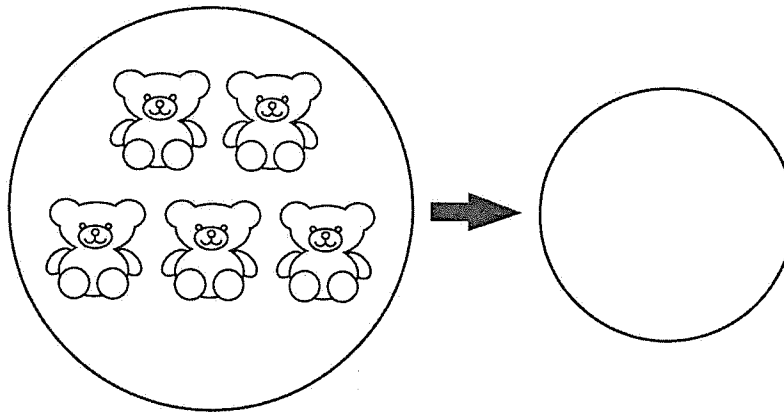
Look Out!

Watch for children who are having a difficult time figuring out which number of bears to move into the take-away (small) circle on the workmat. Remind children that the number of bears that should be placed in the small circle is the number that should be taken away from the group they started with.



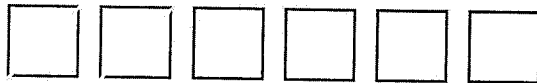
Check children's work.

1.



 2

2.



 4
Directions

1. Five bears are playing on the swings. Three go home. How many bears are left playing on the swings? Use Bear Counters. Model the groups. Write how many are left. 2. Maya had 6 crackers. She ate 2 crackers. Cross out the crackers Maya ate. Write the number of crackers Maya had left.

Answer Key

Check children's work.



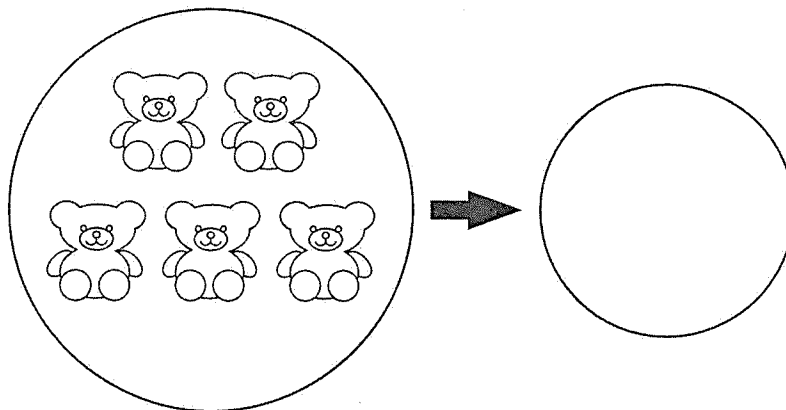
Challenge

There were 5 balls in a box in the gym. The children took 1 ball to play a game. How many balls were left in the box? Draw circles for the balls. Take 1 away. Write how many balls are left in the box.

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1.

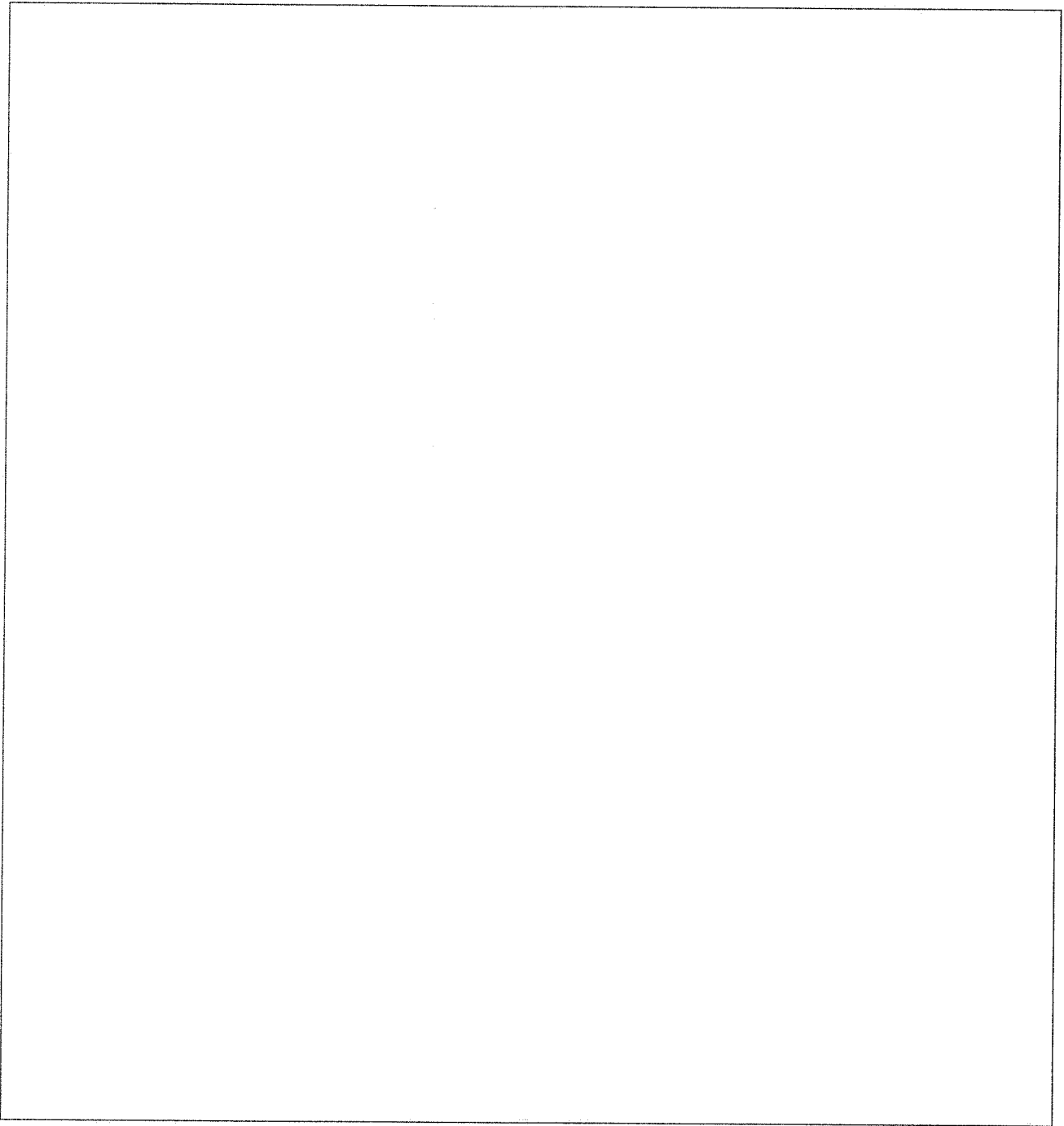


2.

**Directions**

1. Five bears are playing on the swings. Three go home. How many bears are left playing on the swings? Use Bear Counters. Model the groups. Write how many are left. 2. Maya had 6 crackers. She ate 2 crackers. Cross out the crackers Maya ate. Write the number of crackers Maya had left.

Name _____



Challenge

There were 5 balls in a box in the gym. The children took 1 ball to play a game. How many balls were left in the box? Draw circles for the balls. Take 1 away. Write how many balls are left in the box.

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LESSON
3**Objective**

Count and order numbers 0–20.

Common Core State Standards

- 1.NBT.2b The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

Number and Operations in Base Ten**Ordering Numbers**

Counting numbers and ordering them is a requirement for performing most mathematical concepts. A number doesn't mean anything to a child until he or she knows how many the number represents and where it goes on a number line or how it relates to other numbers. Children learn to interpret the count sequence as a list of numbers arranged in order of increasing magnitude. This understanding is a conceptual starting point for comparing numbers and working with the concepts of less than and greater than.

Try It! Perform the Try It! activity on the next page.

Talk About It

Discuss the Try It! activity.

- Have children look at the DecaDots® tiles used in the Try It! activity.
- Ask: Which tile did you put down first? next? last?
- Ask: Which tiles did you replace with a ten tile? What did you do after you had placed the ten tile? Which tiles did you replace with a zero tile?
- Ask: How did you know which tiles were missing?

Solve It

With children reread the problem. Ask children to write a sentence using ordered numbers. Encourage them to say 1 more and 1 fewer in their descriptions.

More Ideas

For other ways to teach about counting and ordering numbers—

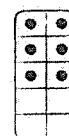
- Have children use Snap Cubes® and number cards to count and order groups to 10. For each number card, have children build a corresponding cube tower. Children then place the cube towers and number cards in order from 0–10.
- Create a number line and use Snap Cubes to count and order groups to 20. Have them arrange their cubes in ascending order in the appropriate number line position.
- Use Cuisenaire® Rods and have students build a staircase with the rods to help them see the correct ordinal position.

Formative Assessment

Have children try the following problem.

Match the pictures to the numbers.

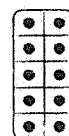
10



6



4



Try It! 30 minutes | Pairs

Here is a problem using ordering.

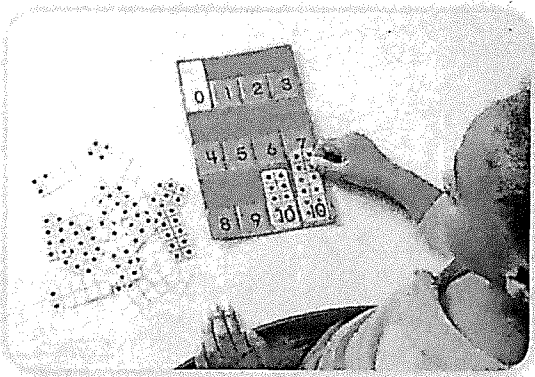
Jeremy has a set of cards labeled 0–20 on his desk. He counts them and notices that three of the cards are missing. How can Jeremy determine which cards are missing?

Introduce the story problem. Then have children do the activity to solve the problem.

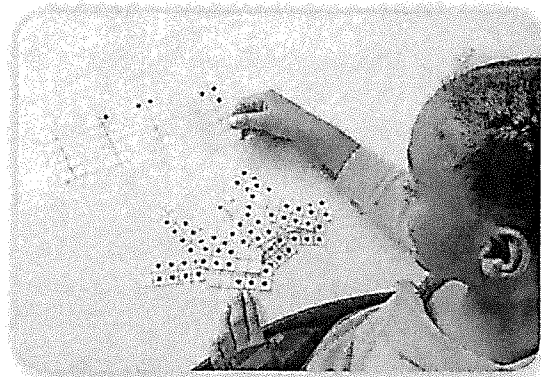
Say: Let's show Jeremy how to determine which cards are missing.

Materials

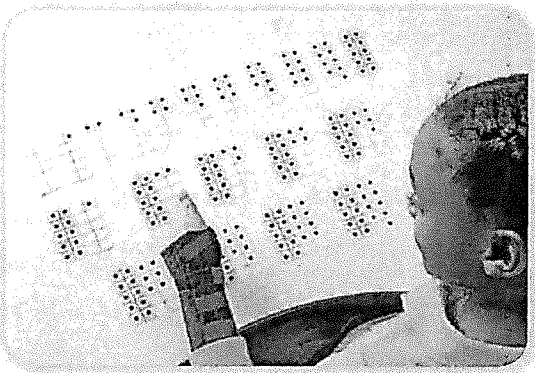
- DecaDots® wallet (1 per pair)



1. To begin, give each pair of children a DecaDots wallet set, and have them remove the tiles.



2. Have children arrange the tiles in a row in ascending order from 0–20. **Ask:** *How do you show a number past 10?*

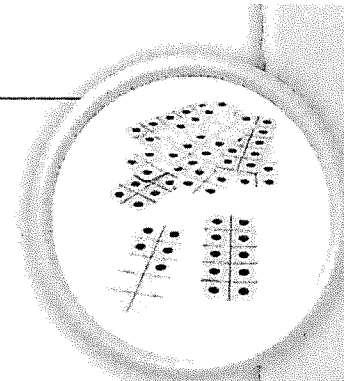


3. Finally, have the children touch each DecaDot as they recite the corresponding number.

Ask: *How would we know if one were missing?*

⚠ Look Out!

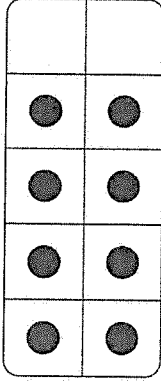
Watch for children who have difficulty placing the numbers in the correct order. Assist these children by placing numbered cards in order and then have them place the corresponding DecaDot tile(s) above the numbered card.



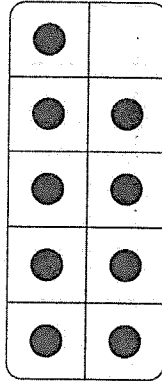
Use DecaDots. Write each number modeled.

Write the three numbers that come next. (Check students' work.)

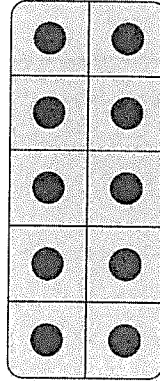
1.



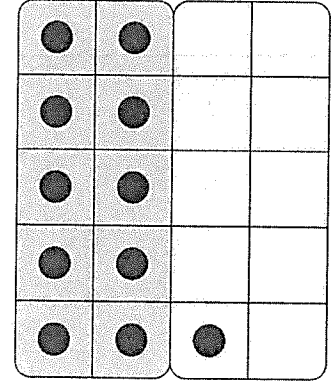
8



9



10



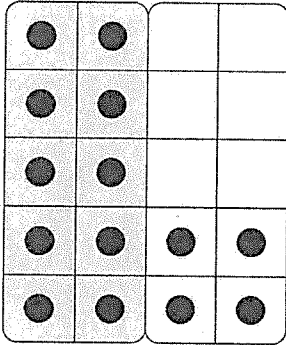
11

Next three numbers: 12 13 14.

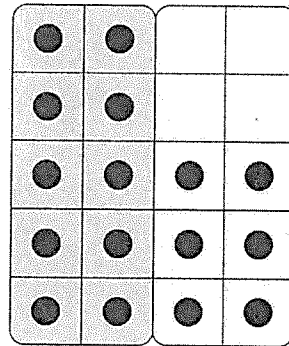
Use DecaDots. Make the missing number.

Draw the model. Write the numbers.

2.



14



16

Missing number: 15



Answer Key

Challenge! What numbers between 0 and 20 use two DecaDots tiles?

Challenge: 11 to 20

Use DecaDots. Write each number modeled.
Write the three numbers that come next.

1.

●	●
●	●
●	●
●	●

●	
●	●
●	●
●	●
●	●

●	●
●	●
●	●
●	●
●	●

●	●		
●	●		
●	●		
●	●		
●	●	●	

Next three numbers: _____

Use DecaDots. Make the missing number.
Draw the model. Write the numbers.

2.

●	●		
●	●		
●	●		
●	●	●	●
●	●	●	●

●	●		
●	●		
●	●	●	●
●	●	●	●
●	●	●	●

Missing number: _____

Name _____

Challenge! What numbers between 0 and 20 use two DecaDots tiles?

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Measurement and Data

Standard Units

When children have had many opportunities to measure using nonstandard units, the transition to standard units is much easier. For less confusion, children should begin measuring in inches, using a ruler without smaller divisions. Estimating lengths can be difficult for children who have little experience measuring because it is hard for them to visualize. However, when children can use a benchmark to base their estimates, the process becomes less abstract.

Objective

Estimate and measure length using standard units.

Common Core State Standards

- 2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
- 2.MD.3 Estimate lengths using units of inches, feet, centimeters, and meters.

Try It! Perform the Try It! activity on the next page.

Talk About It

Discuss the Try It! activity.

- **Ask:** *Did all your measurements come out pretty close?*
- **Say:** *Suppose some groups used Pattern Blocks, some used Snap Cubes®, and others used Inchworms™. Ask: Would the measurements come out just as close as when we all used Inchworms? Why or why not?*
- Discuss the importance of measuring using standard units. **Ask:** *Why might it be important that people all measure things using the same units? Discuss what would happen if people did not have standard units with which to measure.*

Solve It

With children, reread the problem. Have children write Dana a letter telling her how to find out which eraser is longer. Have children discuss using standard units or inches as units in their letters.

More Ideas

For other ways to teach about estimating and measuring length in standard units—

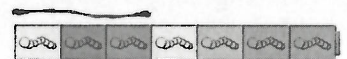
- Have pairs of children estimate the length of a desk in Color Tiles and Inchworms. Then have children measure the desk with both kinds of manipulatives, compare their estimates with the measurements, and compare the measurements.
- Have children use Snap Cubes and Inchworms to measure various classroom objects. Have them first estimate the lengths of the objects and then use the manipulatives to measure them. Have children compare their measurements using the different manipulatives. (Children should conclude that cubes are smaller units than Inchworms.)

Formative Assessment

Have children try the following problem.

How many Inchworms does this string measure?

- A. 6 Inchworms B. 3 Inchworms C. 5 Inchworms



Try It! 30 Minutes | Groups of 3

Here is a problem about estimating and measuring lengths using standard units.

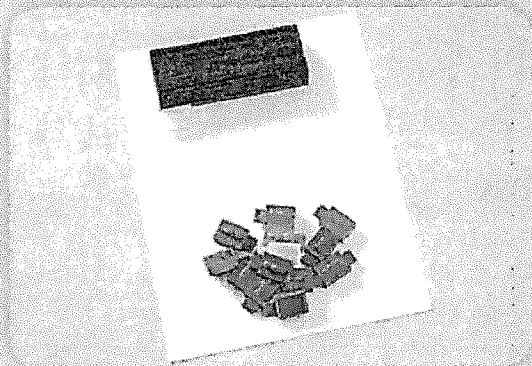
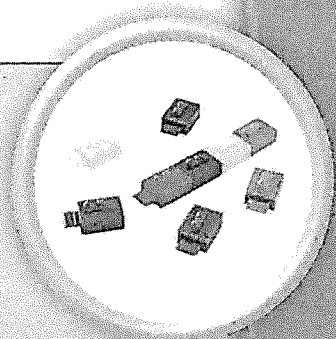
Dana's class is measuring things they find in the classroom. Dana tells her friend Steve that she thinks the board eraser in her room is longer than the eraser in his room. How can she find out if she is right?

Introduce the problem. Then have children do the activity to solve the problem.

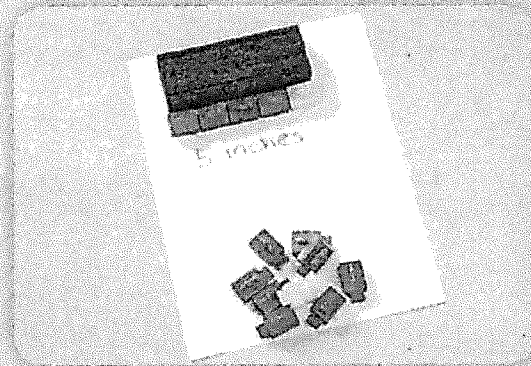
Distribute Inchworms, paper, and crayons to children.

Materials

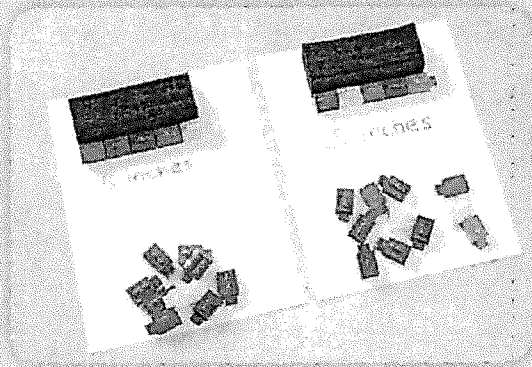
- Inchworms™ (12 per group)
- paper (1 sheet per child)
- crayons (1 per child)



1. Have children hold and carefully examine some Inchworms. **Ask:** *How many Inchworms long do you think the classroom eraser is?* Then have group members discuss their estimates.



2. Next have groups of children take turns using Inchworms to measure the classroom eraser. Have each group member write down his or her measurement.



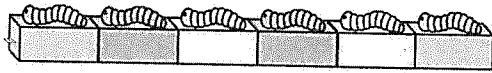
3. Have children compare their estimates to their actual measurements. Then ask each group to tell their actual measurements. Have children clarify that Inchworms, or inches, are being used as units.

⚠ Look Out!

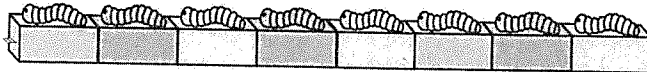
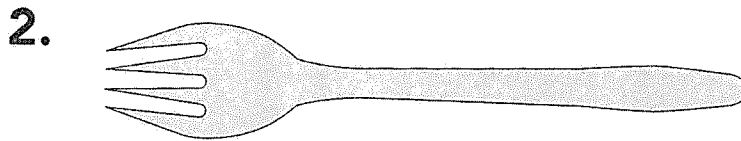
Children might not take care in lining up the Inchworms with the ends of the erasers. Make sure that the Inchworms reach the ends and that they are linked together. Guide children to see how not having the Inchworms or measurement tool aligned will cause inaccurate measurement.

Use Inchworms. Measure each item.

(Check students' work.)



_____ 6 _____ inches



_____ 8 _____ inches

Find each item. Estimate the length.

Use Inchworms to measure the length.

Answers will vary.

3. one side of a book

4. straw

Estimate: _____ inches

Estimate: _____ inches

Actual: _____ inches

Actual: _____ inches

5. crayon

6. dollar

Estimate: _____ inches

Estimate: _____ inches

Actual: _____ inches

Actual: _____ inches



Answer Key

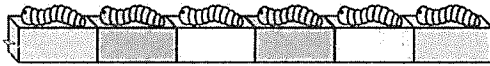
Challenge! How is measuring with Inchworms like measuring with a ruler? How is it different?

Challenge: (Sample) An Inchworm is 1 inch long. Putting 12 Inchworms together end to end is the same as having a 12-inch ruler.


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Use Inchworms. Measure each item.

1. 



_____ inches

2. 



_____ inches

**Find each item. Estimate the length.
Use Inchworms to measure the length.**

3. one side of a book

4. straw

Estimate: _____ inches

Estimate: _____ inches

Actual: _____ inches

Actual: _____ inches

5. crayon

6. dollar

Estimate: _____ inches

Estimate: _____ inches

Actual: _____ inches

Actual: _____ inches

Name _____

Challenge! How is measuring with
Inchworms like measuring with a ruler?
How is it different?

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LESSON
3

Operations and Algebraic Thinking

Exploring Division

Exploring ideas visually and kinesthetically helps students learn new concepts. When students reach grade 3, the emphasis on operations switches from addition and subtraction to multiplication and division. Using concrete models to solve division problems allows students to see the meaning of the different parts of the division problem and how the numbers are tied together.

Objective
Explore the meaning of division.

Common Core State Standards

- 3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.*
- 3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Try It! Perform the Try It! activity on the next page.

Talk About It

Discuss the Try It! activity.

- **Ask:** How many groups of Two-Color Counters did you get when you divided 24 into equal groups of 4? How many groups were there when you divided 24 counters into equal groups of 8?
- **Ask:** What do you notice about the division sentences you wrote when you divided 24 into equal groups of 2 and equal groups of 12? How can you use what you know about multiplication to help you solve a division problem?
- **Ask:** Will you always have equal parts when you divide something? Why or why not?

Solve It

With students, reread the problem. Have students write about how they can use what they know about arrays and multiplication to solve division problems.

More Ideas

For other ways to teach about exploring division—

- Sort Centimeter Cubes and paper cups into evenly divisible groups for a variety of division problems. Present a problem to students, such as *21 divided into equal groups of 3*. Then have students model the problem by placing the correct number of cubes into each cup.
- Have students work in pairs using Color Tiles. One student will make an array using the tiles, and the other student must come up with a multiplication sentence and a division sentence that matches the array. Students take turns creating arrays and multiplication and division sentences.

Formative Assessment

Have students try the following problem.

Which grouping of tally marks shows $24 \div 3$?

- A. ||||| ||||| |||||
- B. ||| ||| ||| ||| ||| |||
- C. ||||| ||||| |||||
- D. ||| ||| ||| ||| ||| ||| ||| |||

Try It! 30 minutes | Pairs

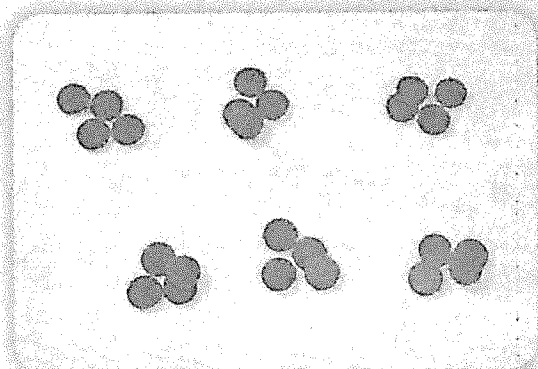
Here is a problem about exploring division.

There are 24 students in Mrs. Lopez's class. Mrs. Lopez divided the class into groups of 4 students. How many groups are there?

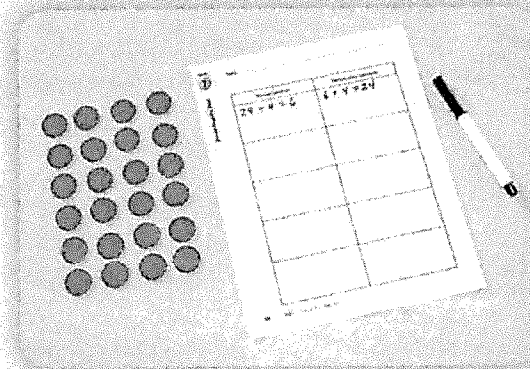
Introduce the problem. Then have students do the activity to solve the problem. Pass out Two-Color Counters and a Division Recording Sheet (BLM 2) to students.

Materials

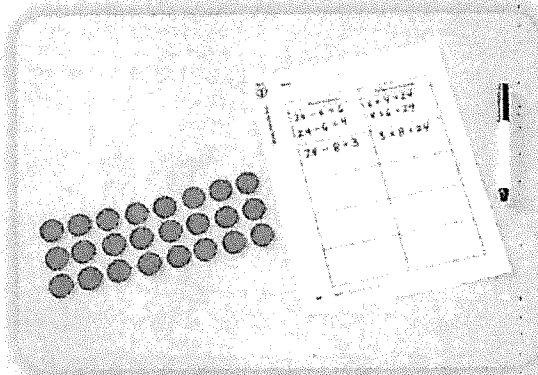
- Two-Color Counters (24 per pair)
- Division Recording Sheet (BLM 2; 1 per student)
- paper (1 sheet per student)
- pencils (1 per student)



1. Say: We are going to divide our counters into equal groups of 4. Tell students that this is one way to show 24 divided into equal groups of 4. **Ask:** What division sentence are we modeling?



2. Have students use their groups to construct an array to show the product of 6 and 4. **Ask:** What multiplication sentence is displayed? Have students fill out the Division Recording Sheet, using counters to assist them.



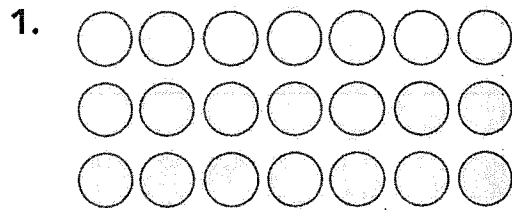
3. Have students use arrays of counters to model the other ways of dividing 24 into equal groups. They should write a division and multiplication sentence for the models they built on the recording sheet.

⚠ Look Out!

If students have difficulty using arrays to perform division, you may wish to show them a multiplication array. Point out that they need 4 columns, and they have 24 counters to use up. Have students put 1 counter in each column, adding rows until the counters have all been used. Also, watch for students who can divide using paper and pencil but cannot display the operation using manipulatives. This may indicate that the student lacks number sense and is relying on the memorization of facts.

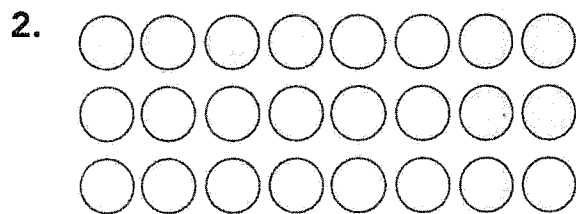
Use Two-Color Counters to build each array. Rearrange the counters into groups of the size shown. Complete each division sentence.

(Check students' work.)



put into groups of

$$\underline{21} \div \underline{3} = \underline{7}$$



put into groups of

$$\underline{24} \div \underline{4} = \underline{6}$$

Build each array using Two-Color Counters. Group the counters to be able to complete each division sentence. (Check students' work.)

3. 45 into 9 groups

4. 32 into 4 groups

5. 30 into 6 groups

$$45 \div \underline{9} = \underline{5}$$

$$32 \div \underline{4} = \underline{8}$$

$$30 \div \underline{6} = \underline{5}$$

Write each division sentence. Write a related multiplication sentence.

6. 15 into 3 groups

$$15 \div \underline{3} = \underline{5}$$

$$\underline{5} \times \underline{3} = 15$$

7. 28 into 7 groups

$$28 \div \underline{7} = \underline{4}$$

$$\underline{4} \times \underline{7} = 28$$

8. 20 into 2 groups

$$20 \div \underline{2} = \underline{10}$$

$$\underline{2} \times \underline{10} = 20$$

9. 35 into 5 groups

$$35 \div \underline{5} = \underline{7}$$

$$\underline{5} \times \underline{7} = 35$$

10. 48 into 8 groups

$$48 \div \underline{8} = \underline{6}$$

$$\underline{6} \times \underline{8} = 48$$

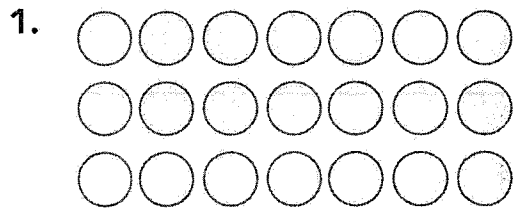
11. 81 into 9 groups


$$81 \div \underline{9} = \underline{9}$$

$$\underline{9} \times \underline{9} = 81$$

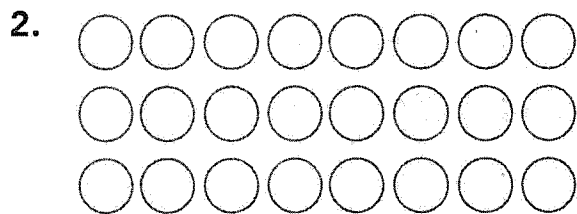


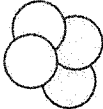
Use Two-Color Counters to build each array. Rearrange the counters into groups of the size shown. Complete each division sentence.



put into groups of 

$$\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$



put into groups of 

$$\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

Build each array using Two-Color Counters. Group the counters to be able to complete each division sentence.

3. 45 into 9 groups

4. 32 into 4 groups

5. 30 into 6 groups

$$45 \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

$$32 \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

$$30 \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

Write each division sentence. Write a related multiplication sentence.

6. 15 into 3 groups

7. 28 into 7 groups

8. 20 into 2 groups

$$15 \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

$$28 \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

$$20 \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

$$\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = 15$$

$$\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = 28$$

$$\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = 20$$

9. 35 into 5 groups

10. 48 into 8 groups

11. 81 into 9 groups

$$35 \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

$$48 \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

$$81 \div \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

$$\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = 35$$

$$\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = 48$$

$$\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = 81$$

Name _____

Challenge! Problem 2 shows three rows of 8 Two-Color Counters for a total of 24 counters. Write a fact family for the model shown. Write a fact family for the model you create from the 24 counters. Explain how the number 24 can have two different fact families.

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Number and Operations—Fractions

Add and Subtract Fractions

Students will need to add and subtract fractions when they begin working with measurements. Sometimes the fractions will have like denominators and other times they will have unlike denominators. In this lesson, students will add and subtract with like denominators.

Objective

Add and subtract fractions with like denominators.

Common Core State Standards

- **4.NF.3a** Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- **4.NF.3d** Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Try It! Perform the Try It! activity on the next page.

Talk About It

Discuss the Try It! activity.

- **Ask:** How do we add or subtract fractions when they have like denominators?
- **Ask:** Can we use subtraction to check our work when adding fractions? How?
- **Ask:** Can we use addition to check our work when subtracting fractions? How?

Solve It

With students, reread the problem. Have students draw a rectangle to represent the mural. Then have them label the sections that the third and fourth graders will paint. Have students write a number sentence for the fraction of the mural that the third and fourth graders are painting and a number sentence for the fraction the fifth graders are painting.

More Ideas

For other ways to teach about adding and subtracting fractions—

- Give students a problem that involves adding or subtracting fractions with like denominators. Students can use Fraction Circles to help them solve the problem.
- Have students grab eight random Color Tiles from a bag or box, then come up with a fraction to represent one of the colors of tiles, such as " $\frac{2}{8}$ of the tiles are red." Then have students use addition and subtraction to model the pile based on that Color Tile, for example, " $\frac{8}{8} - \frac{2}{8} = \frac{6}{8}$ of my group is not red." Challenge students to use addition to check their problems, for example, " $\frac{2}{8} + \frac{6}{8} = \frac{8}{8}$."

Formative Assessment

Have students try the following problem.

Tenysha has $\frac{1}{6}$ of her muffin left. Anna has $\frac{3}{6}$ of her muffin left. Between the two of them, what fraction of a muffin do they have?

- A. $\frac{4}{12}$ B. $\frac{2}{6}$ C. $\frac{4}{6}$ D. $\frac{5}{6}$

Try It! 30 minutes | Groups of 4

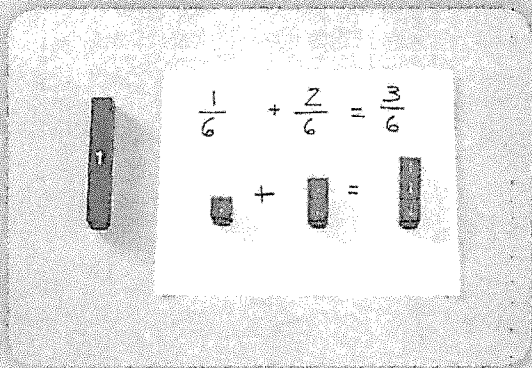
Here is a problem about adding and subtracting fractions.

The third, fourth, and fifth graders at Liberty Elementary School are painting a mural to celebrate school spirit. The third graders will paint $\frac{1}{6}$ of it, and the fourth graders will paint $\frac{2}{6}$ of it. What fraction of the mural will the fifth graders paint?

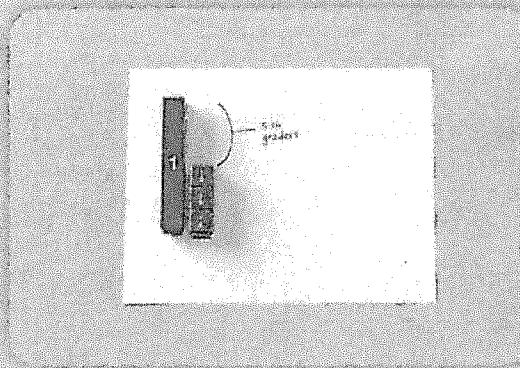
Introduce the problem. Then have students do the activity to solve the problem. Distribute materials to students.

Materials

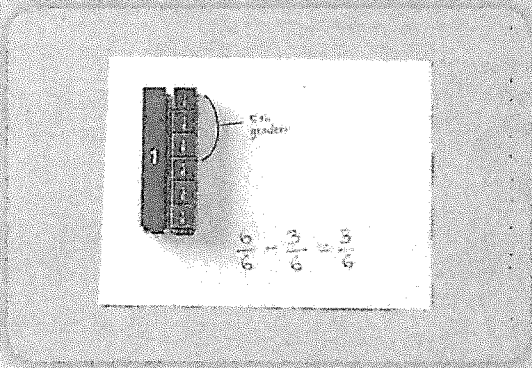
- Fraction Tower® Equivalency Cubes (1 set per group)
- paper (2 sheets per group)
- pencils (1 per group)



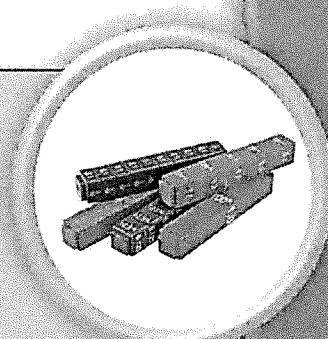
1. Write $\frac{1}{6} + \frac{2}{6}$ on the board. Have students model the problem with one $\frac{1}{6}$ cube and a tower of two $\frac{1}{6}$ cubes. Explain to students that when the denominator is the same, they add the numerators. Have students add the fractions.



2. Elicit that $\frac{1}{6} + \frac{2}{6} = \frac{3}{6}$ is the fraction of the mural painted by the third and fourth graders. **Ask:** How could we represent the fraction painted by the fifth graders? Have students place a $\frac{3}{6}$ tower next to the whole and elicit that the difference between the whole and the $\frac{3}{6}$ tower is the fraction painted by the fifth graders.




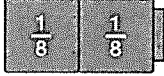
3. **Ask:** What is the difference? Let students find that a $\frac{3}{6}$ tower completes the whole, so the difference is $\frac{3}{6}$. Have students write the difference as a number sentence.








Look Out!

Students might want to represent the section of the mural the fifth graders paint as $\frac{1}{2}$. Explain that $\frac{1}{2}$ is correct, but that in this lesson they are working with like denominators, so they should work with sixths. Tell students they will work with unlike denominators in the future.

Use Fraction Towers to model each sum or difference. Write the fractions and the answer. (Check students' work.)

1.  +  = $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$

2.  -  = $\frac{2}{5} - \frac{1}{5} = \frac{1}{5}$

3.  +  +  = $\frac{6}{12} + \frac{1}{12} + \frac{4}{12} = \frac{11}{12}$

Using Fraction Towers, model each sum or difference. Then sketch the models below. Complete the equation.

4. $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$

5. $\frac{6}{8} - \frac{3}{8} = \frac{3}{8}$

6. $\frac{2}{10} + \frac{7}{10} = \frac{9}{10}$

(Check students' models.)

Find each sum or difference.

7. $\frac{3}{12} + \frac{7}{12} = \frac{10}{12}$

8. $\frac{9}{10} + \frac{2}{10} = \frac{11}{10}$

9. $\frac{8}{12} + \frac{2}{12} + \frac{5}{12} = \frac{15}{12}$

10. $\frac{8}{10} - \frac{5}{10} = \frac{3}{10}$

11. $\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$

12. $\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$

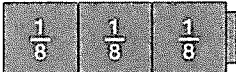
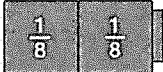




Answer Key

Challenge! Show $\frac{3}{4} + \frac{3}{4}$ on a number line.

Challenge: Check drawings. Students should mark off 2 intervals, 0 to $\frac{3}{4}$ and $\frac{3}{4}$ to $1\frac{2}{4}$, or $1\frac{1}{2}$.

Use Fraction Towers to model each sum or difference. Write the fractions and the answer.

1.  +  + _____ = _____

2.  -  - _____ = _____

3.  +  +  + _____ = _____

Using Fraction Towers, model each sum or difference. Then sketch the models below. Complete the equation.

4. $\frac{2}{5} + \frac{1}{5} =$ _____

5. $\frac{6}{8} - \frac{3}{8} =$ _____

6. $\frac{2}{10} + \frac{7}{10} =$ _____

Find each sum or difference.

7. $\frac{3}{12} + \frac{7}{12} =$ _____

8. $\frac{9}{10} + \frac{2}{10} =$ _____

9. $\frac{8}{12} + \frac{2}{12} + \frac{5}{12} =$ _____

10. $\frac{8}{10} - \frac{5}{10} =$ _____

11. $\frac{4}{5} - \frac{1}{5} =$ _____

12. $\frac{5}{6} - \frac{2}{6} =$ _____

Name _____

Challenge! Show $\frac{3}{4} + \frac{3}{4}$ on a number line.

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Number and Operations—Fractions

Subtract Fractions with Unlike Denominators

Objective

Subtract fractions with unlike denominators.

Common Core State Standards

- 5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$.)
- 5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$.

Students model the subtraction of fractions with unlike denominators as the first step toward using an algorithm to subtract. Many students will use prior experience with finding equivalent fractions to find common denominators before subtracting. Students also may use number sense or reasoning to find the solution to a problem.

Try It! Perform the Try It! activity on the next page.

Talk About It

Discuss the Try It! activity.

- **Ask:** When finding the difference between two fractions, why can you subtract the numerators of two fractions with the same denominator, but cannot subtract the numerators of two fractions with unlike denominators?
- **Ask:** Which fraction or fractions would you rename to find $\frac{9}{10} - \frac{2}{5}$?
- Have students compare addition and subtraction of fractions with unlike denominators. **Ask:** How can you use addition to check your answer to a subtraction problem?

Solve It

Reread the problem with students. Have students explain in writing why they needed to subtract to find the solution. Then have them describe how they found the quantity of milk needed from the second carton.

More Ideas

For other ways to teach subtracting fractions with unlike denominators—

- Have students use Deluxe Rainbow Fraction® Circles or Fraction Tower® Equivalency Cubes to model subtraction problems.
- Have students use the hexagon, blue rhombus, trapezoid, and triangle from a set of Pattern Blocks to model subtraction. Let the hexagon represent one whole; the blue rhombus, $\frac{1}{3}$; the trapezoid, $\frac{1}{2}$; and the triangle, $\frac{1}{6}$. Have students write as many subtraction sentences as they can, using these fractions. Suggest that students fit pieces over larger pieces to help them find each difference.

Formative Assessment

Have students try the following problem.

Jordan and Mark are painting opposite sides of a fence. Mark has painted $\frac{7}{10}$ of his side. Jordan has painted $\frac{1}{2}$ of his side. How much more has Mark painted than Jordan?

- A. $\frac{5}{8}$ B. $\frac{5}{10}$ C. $\frac{1}{5}$ D. $\frac{1}{10}$

Try It! 15 minutes | Groups of 4

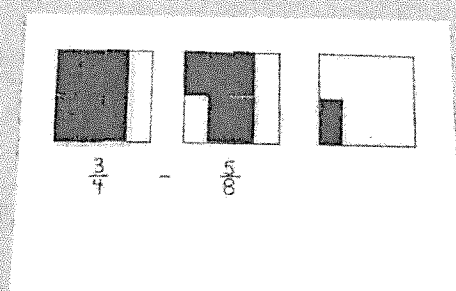
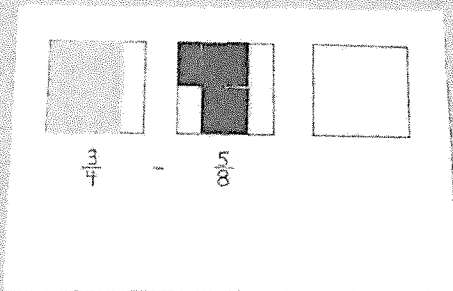
Here is a problem about subtracting fractions with unlike denominators.

A cornbread recipe calls for $\frac{3}{4}$ cup of milk. Rachel uses the last $\frac{5}{8}$ cup of milk in one carton. She opens another carton and pours the remaining amount needed. How much milk does Rachel use from the newly opened carton?

Introduce the problem. Then have students do the activity to solve the problem. Distribute Fraction Squares, paper, and pencils to students. Have students trace 3 whole squares in a row on the paper.

Materials

- Deluxe Rainbow Fraction® Squares (2 sets per group)
- paper (11" x 17"; 1 sheet per group)
- pencils (1 per group)

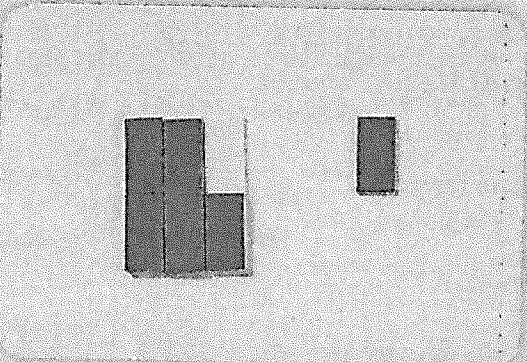


1. Have students model each fraction.

Ask: What expression can you write to show the situation? Write $\frac{3}{4} - \frac{5}{8}$ on the board.

Ask: Using what you know about adding two fractions, what do you think you should do first to subtract these two fractions?

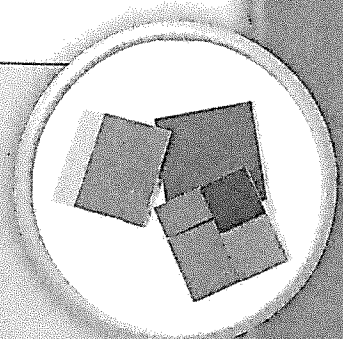
2. Have students substitute $\frac{6}{8}$ for $\frac{3}{4}$. **Ask:** Now that both fractions have the same denominator, how can you find the difference? Have students write and model the subtraction equation for the situation.



3. **Say:** The Fraction Squares can be used to check your answer. Have students model each fraction again. This time have them place the five blue pieces on top of the three yellow pieces. **Ask:** What piece is needed to completely cover the three yellow pieces? What fraction does this piece represent? Is this fraction the same as your answer?

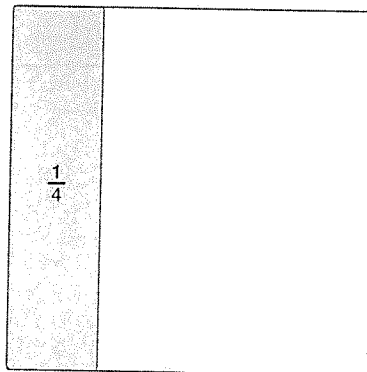
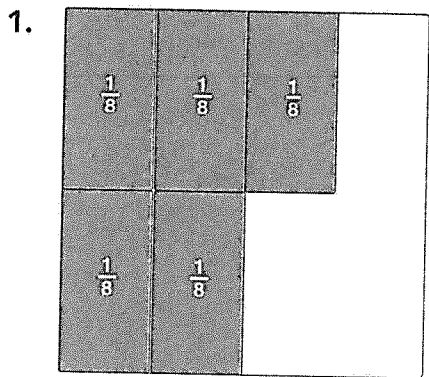
Look Out!

Some students may not measure the squares properly when finding equivalent fractions. Suggest that these students place the Fraction Squares over each other to ensure that the fractional values are exactly the same. Other students may not recognize the importance of finding common denominators, instead looking for a Fraction Square that will be the same size. Have these students explain why their solution works and how they can use it to find the differences in other situations.



Use Fraction Squares to model the fractions shown. Use Fraction Squares to find fractions with the same denominators. Write the fractions and then find the difference.

(Check students' work.)



$$\underline{\frac{5}{8}} \quad - \quad \underline{\frac{1}{4}} \quad = \quad \underline{\frac{3}{8}}$$

Using Fraction Squares, model the subtraction problem. Sketch the model. Write the difference.

(Check students' models.)

2. $\frac{5}{6} - \frac{1}{3}$

3. $\frac{3}{4} - \frac{3}{8}$

$$\underline{\frac{1}{2}}$$

$$\underline{\frac{3}{8}}$$

Find each difference.

4. $\frac{3}{4} - \frac{5}{12} = \underline{\frac{1}{3}}$

5. $\frac{5}{6} - \frac{2}{3} = \underline{\frac{1}{6}}$

6. $\frac{3}{5} - \frac{1}{10} = \underline{\frac{1}{2}}$

7. $\frac{3}{4} - \frac{1}{12} = \underline{\frac{2}{3}}$

8. $\frac{5}{8} - \frac{1}{4} = \underline{\frac{3}{8}}$

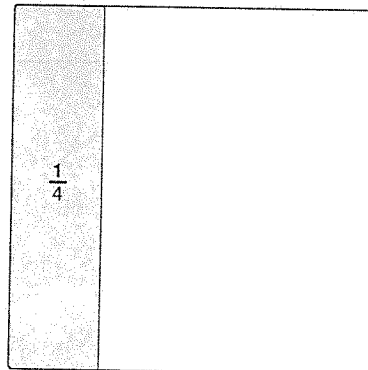
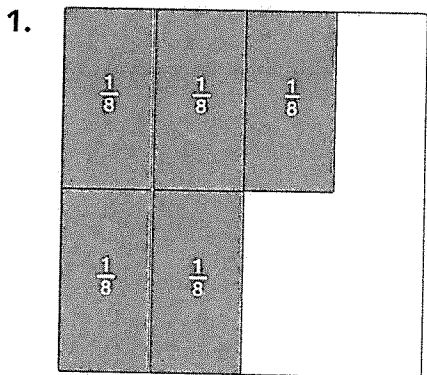
9. $\frac{2}{5} - \frac{1}{10} = \underline{\frac{3}{10}}$

10. $\frac{2}{3} - \frac{5}{12} = \underline{\frac{1}{4}}$

11. $\frac{7}{12} - \frac{1}{4} = \underline{\frac{1}{3}}$



Use Fraction Squares to model the fractions shown. Use Fraction Squares to find fractions with the same denominators. Write the fractions and then find the difference.



_____ - _____ = _____

Using Fraction Squares, model the subtraction problem. Sketch the model. Write the difference.

2. $\frac{5}{6} - \frac{1}{3}$

3. $\frac{3}{4} - \frac{3}{8}$

Find each difference.

4. $\frac{3}{4} - \frac{5}{12} =$ _____

5. $\frac{5}{6} - \frac{2}{3} =$ _____

6. $\frac{3}{5} - \frac{1}{10} =$ _____

7. $\frac{3}{4} - \frac{1}{12} =$ _____

8. $\frac{5}{8} - \frac{1}{4} =$ _____

9. $\frac{2}{5} - \frac{1}{10} =$ _____

10. $\frac{2}{3} - \frac{5}{12} =$ _____

11. $\frac{7}{12} - \frac{1}{4} =$ _____



